

The Earth rotates from west to east, but the sun, moon, stars and planets rise in the east and set in the west. In this exercise you will prove that this must happen using geometric skills. Follow the Construction instructions in Part 1 to draw the figure and label the relevant points, line segments and angles, then answer the accompanying questions and construct the Proof.

**Construction:**

- 1 Draw a circle with center O
- 2 Draw the vertical radius OA
- 3 Draw radius OB to the right of OA such that AOB is an acute angle
- 4 Draw a ray tangent to radius OA
- 5 Draw a ray tangent to radius OB intersecting the tangent ray to OA at point C
- 6 Extend radius OB so that it intersects the tangent to radius OA at point D
- 7 Draw segment DF where point F is on the tangent to radius OB
- 8 Draw a ray, parallel to AD that intersects the circle at point B, and the segment DF at point E
- 9 Extend ray BE so that it intersects radius AO at point G

**Givens:**

AD parallel to GE

CD perp DE

BE perp DF

**Problem 1:** Prove Triangle ADO is congruent to Triangle CDF

**Problem 2:** Prove Angle AOB equals angle EBF

**Problem 3:** Draw a second diagram similar to the one in Problem 1 but in which  $m\angle AOB$  is increased. Re-perform the Proof in Problem 1. Prove that, as  $m\angle AOB$  increases that  $m\angle EBF$  remains equal to  $m\angle AOB$  and increases as well.

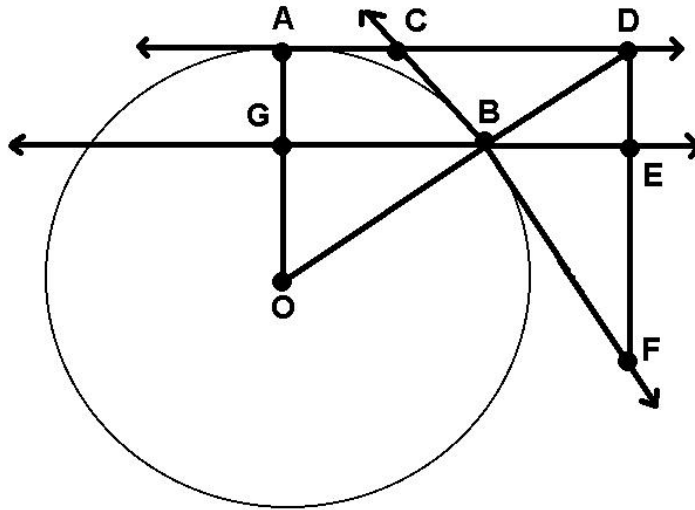
Suppose that the circle represents Earth, and you are looking down at it from above the north pole. Earth rotates from west to east in the diagram. Suppose that the tangent line CF represents the horizon line for Observer B, and that a few hours later, Earth has rotated eastwards so that Observer B is now located at Point A with a local horizon defined by ray AD. Suppose that Observer B sees the sun located along the ray direction BE

**Problem 4:** Prove that the direction of the sun from Observer Bs location is in the western sky.

**Problem 5:** Prove that the direction of the sun after a few hours at Point A is at the western horizon.

**Problem 6:** Explain how the observation that objects rise in the east and set in the west require that Earth rotate from west to east.

**Goal:** Students will use geometric knowledge to prove that, though Earth rotates from west to east, the sun, moon, stars and planets will appear to rise in the east and set in the west.



**Part 1:** Students will follow the step-by-step instructions in the Construction Phase and draw the figure to the left.

**Problem 1:** Prove Triangle ADO is congruent to Triangle CDF. **Answer:** Angle OAD = Angle CDF = right angle; ADB congruent to DBE; BDE congruent to EBF; BEF = right angle; EBF congruent to BFE; so BFE = CDB; so AOD = BDF; and so Triangle ADO congruent to CDF. Students should provide supporting postulate citations for each step.

**Problem 2:** Prove Angle AOD equals angle EBF. **Answer:** Angle ADO is congruent to Angle BFD. Since BEF is a right angle, EBF must be equal to ADO.

**Problem 3:** Prove that, as  $\angle AOB$  increases that  $\angle EBF$  increases as well. **Answer:** Students draw a second diagram, but one in which  $\angle AOB$  is larger, then perform the proof in Problem 1 over again to demonstrate, by deduction, that  $\angle AOB = \angle EBF$  so that again their measures remain equal.

**Problem 4:** Prove that the direction of the sun from Observer B's location is in the western sky. **Answer:** This requires visual thinking. Given the information provided by the list of suppositions, the two parallel rays represent the light from a star fixed at infinity in the direction BE (ie the sun). This direction is to the west as viewed by someone standing on the Earth at point B.

**Problem 5:** Prove that the direction of the sun after a few hours at Point A is at the western horizon. **Answer:** The observer is now at point A since the circle has rotated counterclockwise from west to east. The ray AD still points to the sun 'at infinity', which is now on the western horizon defined by the tangent line AD at point A.

**Problem 6:** Explain how the observation that objects rise in the east and set in the west require that Earth rotate from west to east. **Answer:** Because if the opposite were true, the object in the sky would appear to rise in the west and set in the east, as the observer moved from Point A to point B.